MATH1070 Prac 1: Chaos

The Logistic Map

In this prac you will:

- Use MATLAB to iterate different initial conditions for the logistic map for different values of `r`.
- Draw trajectories in phase space using printed plots from MATLAB.
- Create a bifurcation diagram for the logistic map using nested loops in MATLAB.

Most of this work will be assessed in the first assignment.

Task 1: Dynamics

1. Write an m-file to iterate the logistic map: $x(t+1) = r\,x(t)\,(1-x(t))$. You should use a 'for' loop and may want to make it a function so that you can easily set the initial population size, the growth rate and the number of time steps. I suggest preallocating space for the variable `x` with something like:
   ```
   x=zeros(1,stop);
   ```
   This makes the program run faster and will be important later.

2. Pick a value of `r` less than 2.8 and an initial `x` in (0,1). Iterate the map until it ‘settles down’ and plot `x` over time. Label the axes and add a title that includes the value of `r` you chose.

3. Now choose a different initial condition in (0,1), iterate the map for the same number of time steps and add this new data to your plot with a different colour and/or marker (you will need to use the command `hold on`). Add a legend (easiest to use Insert → Legend on the figure then edit it) and label the data with the appropriate initial condition, something like `x(1)=0.23`.

   Describe what happens to the dynamics in response to the different initial conditions.

4. Create another figure as before, use `r=3.2` and two different initial conditions of your choosing.

   Describe what happens to the dynamics in response to the different initial conditions this time.

5. Create another figure as before, use `r=3.9` and choose two different initial conditions that are 0.0001 apart. Iterate the system for 60 time steps and describe what happens this time.

Task 2: Phase Space

1. Plot the line `y=r*x*(1-x)` for `r=2.5` and the line `y=x` for x in (0,1). Turn the grid lines on and set both axes to show the interval (0,1).
2. Print the plot.
3. Print out two more of the same plots, one for `r=3.2` and one for `r=3.9`
4. Pick any initial condition and use a pencil to trace the phase space trajectory and determine the behaviour of these maps.

Task 3: Bifurcation Diagram

To create a bifurcation diagram, we're going to use 4000 different values of `r` and iterate the map 500 times for each value from the same initial condition. We will ignore the first 250 iterations as the *burn in* and plot the last 250 values as points for each value of `r`. The `x` values will go on the y-axis and `r` will be on the x-axis.

1. Write an m-file to create the data for the bifurcation diagram. I recommend using two nested `for` loops (one inside the other). After you set up the initial conditions and preallocate `x` as a 4000 by 500 matrix, the outer `for` loop should increment `r` by 0.001 from 0.001 to 4. The inner loop should iterate the map for 500 time steps with the current value of `r`.
2. Finally, you will need to plot the last 250 values of `x` for each value of `r`. I recommend using small black dots (`'k.','MarkerSize',4`).
3. Label the plot.
4. Plot the data again and zoom in: change the `r` axis to go from 3 to 4.

Task 4: Analysis

1. Find the fixed point of the linear map: $x(t+1) = f(x(t)) = a + b\,x(t)$ and show that the period-2 values, $x(t+2) = x(t)$, are actually the same as the period-1 point.

   Hint, let $x(t+2) = x(t) = x^*$. Since $x(t+2) = f(x(t+1)) = f(f(x))$, solve $x^* = f(f(x^*))$.

   Under what conditions is the period-1 fixed point stable?

   Advanced: can you work out the closed form solution to the linear map?
2. Find both fixed points of the logistic map.

   For what values of r do the two fixed points exist in [0,1]?

   How does the stability of each point change with r?

   Find the period-2 values.

   What equation do you have to solve to find the period three values?